Acoustic band gaps created by rotating square rods in a two-dimensional lattice

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Acoustic band gaps can be opened and tuned by rotating square rods in two-dimensional liquid sonic crystals. For the systems of mercury rods with square cross section in a water host, the width of lowest gaps increases as the rotation angle of the square rods increases. But opposite results are found for the inverse systems of water rods in mercury, where the lowest gaps narrow with an increase in the rotation angle. This gap-tuning effect becomes more evident with the filling fraction increase. Such an effect should open up a new way for designing acoustic band gaps in two-dimensional phononic crystals.

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I. INTRODUCTION

In the past few years, there has been growing interest in the propagation of acoustic or elastic wave in periodic elastic composite materials termed phononic crystals [1–13]. Such artificial crystals can exhibit acoustic/elastic band gaps in which sound and vibration are all forbidden in any direction. The existence of acoustic/elastic band gaps is significant for better understanding the Anderson localization of sound and vibrations in composite media [14], as well as their numerous engineering applications such as acoustic/elastic filters, vibrationless environments for high-precision mechanical systems, and improvements in the design of new transducers.

Several methods have been used to study the acoustic/elastic band structures, such as, the plane-wave expansion (PWE) method [2–4], the finite difference time domain (FDTD) method [5–7], the multiple-scattering theory (MST) [1,8–10], and perturbative approach [15]. Various structures of two-dimensional (2D) phononic crystals were investigated. Large acoustic band gaps in square, triangular, boron nitride, and rectangular lattices [2–4,15–17] have been found, the constituent materials being either solids, or both fluids (gases), or mixed solid-fluid [10–13,15–17]. Since the superior features of phononic crystals result from the phononic band gap, it is essential to design crystal structure with a band gap as large as possible. Caballero *et al.* [17] have studied the two-dimensional square and triangular lattices of rigid cylinders in air. By reducing the structure symmetry, the absolute sonic band gaps can be enlarged. Recently Yun Lai *et al.* [15] reported that the acoustic band gap can be enlarged or reduced by altering the microstructure for the square lattice of steel cylinders in an air background, and of mercury (water) rods in a water (mercury) host.

The present work is motivated by the similar works in photonic crystals [18] and phononic crystals [19]. Wang *et al.* [18] have pointed out that the full photonic band gap could be enlarged by rotating noncircular (square, hexagonal, double hybrid, double circular) air rods in dielectric background. Goffaux *et al.* [19] have studied the 2D phononic band gap system consisting of solid and air. In the system of the square solid rods in air, the PWE method can be approximately used well, because of the high-density contrast between solid and air. The results show that the acoustic gap width increases progressively with the rotation angle increasing for each filling fraction and, at a fixed angle, increases with filling fraction increasing. However, they only considered the case of low solid filling fraction $F \leqslant 0.50$, and pointed out that 45° seems to be the best angle for generating the largest gap. But in our opinion the air modes for high solid filling fraction ($F \geqslant 0.50$) can be also calculated by the PWE method as long as $\theta \leqslant \theta_c$, where θ_c is the limit value of the rotation angle. Moreover, the acoustic gaps for high solid filling fraction ($F \geqslant 0.50$) will be larger than that largest gap calculated by Goffaux *et al.* [19]. For the opposite case of square air rods in solid host, in order to use the PWE method to calculate the elastic modes, they introduce an artificial transverse velocity into air. The results show that the gaps of the elastic modes (transverse and mixed cases) also increase as the rotation angle enlarges. In this paper, we consider the 2D liquid phononic crystals composed of water (with longitudinal velocities $c_l = 1.48$ km/s and density $\rho = 1.0 \times 10^3$ kg/m^3) and mercury ($c_l = 1.45$ km/s, $\rho = 13.5 \times 10^3$ kg/m^3). Both water and mercury have almost the same wave velocity, and at the same time have high-density contrast. The advantage of the system lies in that the PWE method can be used exactly to calculate the acoustic band gaps for both the system of mercury in water and the system of water in mercury, without consideration of any limit of rotation angle for a large filling fraction. The results show that the lowest gaps increase monotonically as the rotation angle increases for the systems of mercury square rods in a water host, and the largest band gap is attained at high filling fraction with the close-packed limit. But for the system of water square rods in a mercury host, an opposite result is found; the lowest gaps will decrease monotonically as the rotation angle increases.

The paper is organized as follows. The method and model are briefly presented for fluid-fluid binary 2D composite media in Sec. II. We discuss the numerical results in Sec. III, and draw conclusions in Sec. IV.

II. METHOD AND MODEL

Now we calculate the acoustic band structure by using the plane-wave expansion (PWE) method. In the 2D periodic

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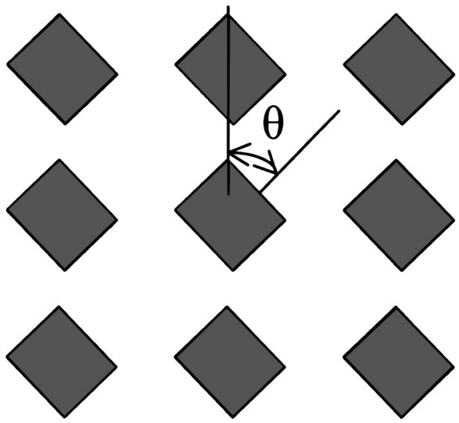

FIG. 1. A transverse cross section of the 2D square lattice. The two constituents (A, B) are denoted by the black and white sectors, respectively. The filling fraction for component A is $F=(2l/a)^2$, where $2l$ is the edge length of the square rods, and a is the lattice constant. The model is periodic in the x and y directions but homogeneous in the z direction.

composite media of liquid, only the longitudinal waves are allowed. The acoustic wave equation can be simplified as follows [2,16]:

$$\frac{1}{\lambda}\frac{\partial^2 p}{\partial t^2}=\nabla\left(\frac{\nabla p}{\rho}\right), \tag{1}$$

where $\lambda(\vec{r})$, $\rho(\vec{r})$, and p are the bulk modulus, the mass density, and pressure of the fluid, respectively.

The pressure p in Eq. (1) can be written by the Bloch theorem as follows:

$$p(\vec{r})=e^{i\vec{k}\cdot\vec{r}}p_{\vec{k}}(\vec{r}), \tag{2}$$

where $\vec{k}$ is restricted within the first Brillouin zone (BZ) and is a periodic function with the same periodic structure as $1/\lambda(\vec{r})$ and $1/\rho(\vec{r})$. They can be expanded in Fourier series:

$$f(\vec{r})=\sum_{\vec{G}} f_{\vec{G}}e^{i\vec{G}\cdot\vec{r}}, \tag{3}$$

where $f(\vec{r})$ stands for $1/\lambda(\vec{r})$, $1/\rho(\vec{r})$, or $p_{\vec{k}}(\vec{r})$, and the summation extends over all the reciprocal vectors $\vec{G}$. Substituting them into Eq. (1) yields

$$\sum_{\vec{G}'}[\omega^2\lambda^{-1}_{\vec{G}-\vec{G}'}-\rho^{-1}_{\vec{G}-\vec{G}'}(\vec{K}+\vec{G})\cdot(\vec{K}+\vec{G}')]P_{\vec{G}'}=0. \tag{4}$$

Just as pointed out by Sigalas *et al.* [3], it is interesting that we can achieve much better convergence with Eq. (4) rather than the equivalent wave equation where λ appears in the right part of the equation. The Fourier coefficients $f_{\vec{G}}$ can be expanded and simplified for a binary composite as follows:

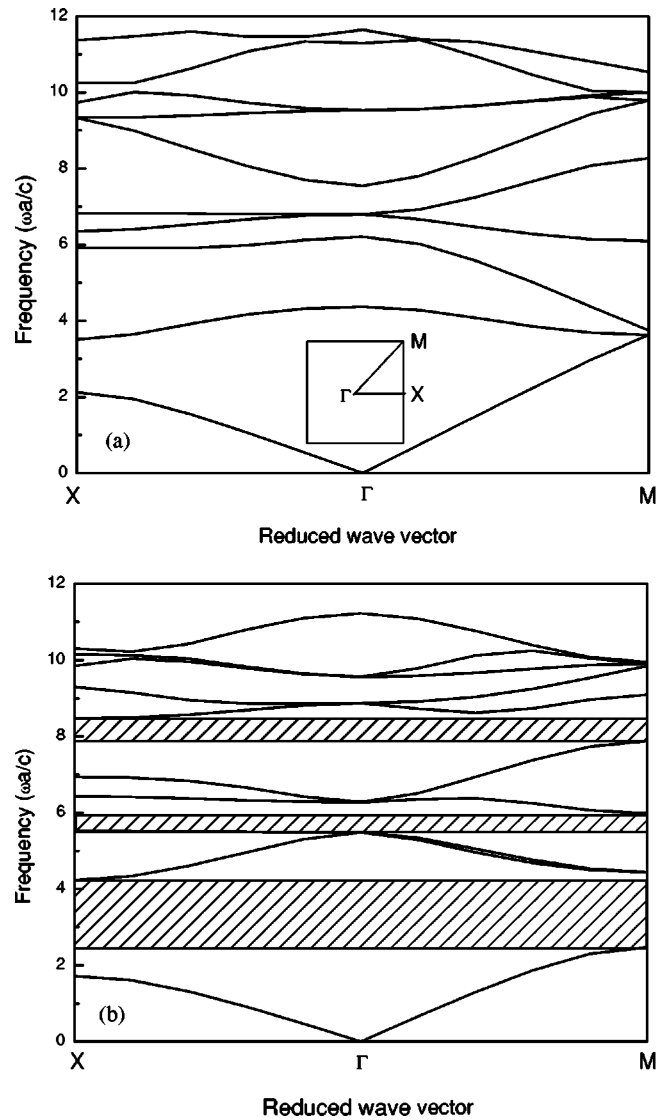

FIG. 2. Acoustic band structures for mercury cylinders in a water host. The filling fraction is $F=0.50$: (a) $\theta=0°$ and (b) $\theta=45°$. The inset shows the first Brillouin zone of the two-dimensional square lattice. In (b) the hatched region represents a complete band gap.

$$f_{\vec{G}}=f_A+(1+F)f_B, \quad \vec{G}=\vec{0},$$

$$\text{or}\quad f_{\vec{G}}=(f_A-f_B)I(\vec{G}), \quad \vec{G}\neq\vec{0}, \tag{5}$$

where A and B denote the rods and background, respectively. For the system of square rods with rotation angle θ (see Fig. 1), we can easily obtain the structure factor $I(\vec{G})$ as [18]

$$I(\vec{G})=F\sin\left(\frac{\widetilde{G_x}l}{\widetilde{G_x}l}\right)\sin\left(\frac{\widetilde{G_y}l}{\widetilde{G_y}l}\right), \tag{6}$$

$$\widetilde{G_x}=G_x\cos\theta+G_y\sin\theta,$$

$$\widetilde{G_y}=-G_x\sin\theta+G_y\cos\theta, \tag{7}$$

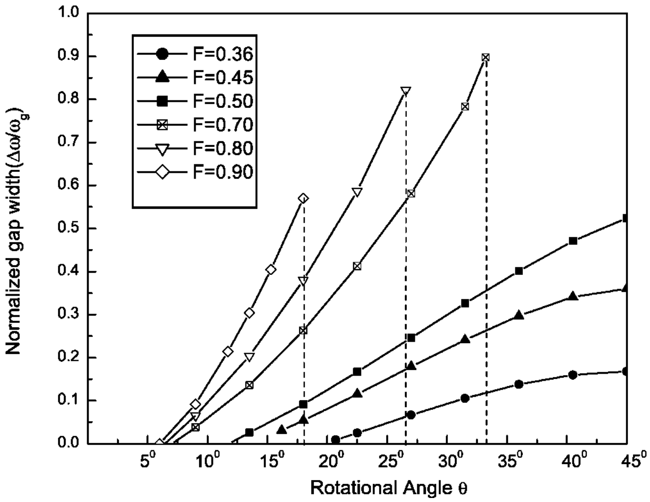

FIG. 3. The normalized gap width of the lowest band gap as the function of the rotation angle for five different filling fractions $F = 0.36$, 0.45, 0.50, 0.58, and 0.72, respectively.

where $2l$ is the width of square cross section of rods, and $F = 4l^2/a^2$ is the filling fraction of rods. In the present article, we have used 625 reciprocal vectors to do the calculation. The results have shown a very good convergence.

III. RESULTS AND DISCUSSION

First, we investigate the case of mercury rods in water host. The acoustic band structures are shown in Figs. 2(a) for $\theta = 0°$ and 2(b) for $\theta = 45°$ at the filling fraction of $F = 0.50$, respectively. The absolute acoustic band gap does not exist at $\theta = 0°$ in the first ten bands, as shown in Fig. 2(a). More detailed numerical calculations also show that there is no absolute acoustic band gap between the first two bands for any filling fraction at this orientation [16]. The configuration can be changed by rotating the square rods. As shown in Fig. 2(b), there are three band gaps among the first ten bands. The degeneracy of the first two bands at M point is lifted; this opens the lowest band gap which is the largest one among the three band gaps with gap width $\Delta\omega = 1.76c/a$ and normalized gap width $\Delta\omega/\omega_g = 0.52$, where ω_g is the midgap frequency.

Figure 3 shows the numerical results of the normalized gap width $\Delta\omega/\omega_g$ of the lowest band gap as a function of the rotation angle θ for six different filling fractions $F = 0.36$, 0.45, 0.50, 0.70, 0.80 and 0.90 in turn. Considering the similarity between the cases of $\theta \geqslant 45°$ and $\theta \leqslant 45°$, we only give the results of the $\theta \leqslant 45°$ case. One can easily find that the gaps only appear in a certain range of rotation angle θ, the gaps open at $\theta = 6.1°$, 6.48°, 7.60°, 12.04°, 14.10°, 20.48°, respectively, which enlarge gradually as the filling fraction increases. Meanwhile, the gaps also enlarge as the rotation angle θ increase. For filling fraction $F = 0.36$, 0.45, and 0.50, the gaps all reach their maxim at the same angle $\theta = 45°$. But for filling fraction $F = 0.7$, 0.80, 0.90, the gaps attain to the maxim at their corresponding closed-packing angles, i.e., $\theta_c = 33.17°$, 26.68°, and 17.91°, with normalized gap width $\Delta\omega/\omega_g = 0.90$, 0.82, and 0.57, respectively. Noticeable is the fact that, when $F = 0.70$, the gap appears in the largest rotation angle θ range from 7.60° to 33.17° as well as the largest normalized gap width $\Delta\omega/\omega_g = 0.90$. The angle range and the maximum gaps narrow gradually when $F \geqslant 0.70$, while the angle range and maximum gaps broaden when $F \leqslant 0.70$ as F increases.

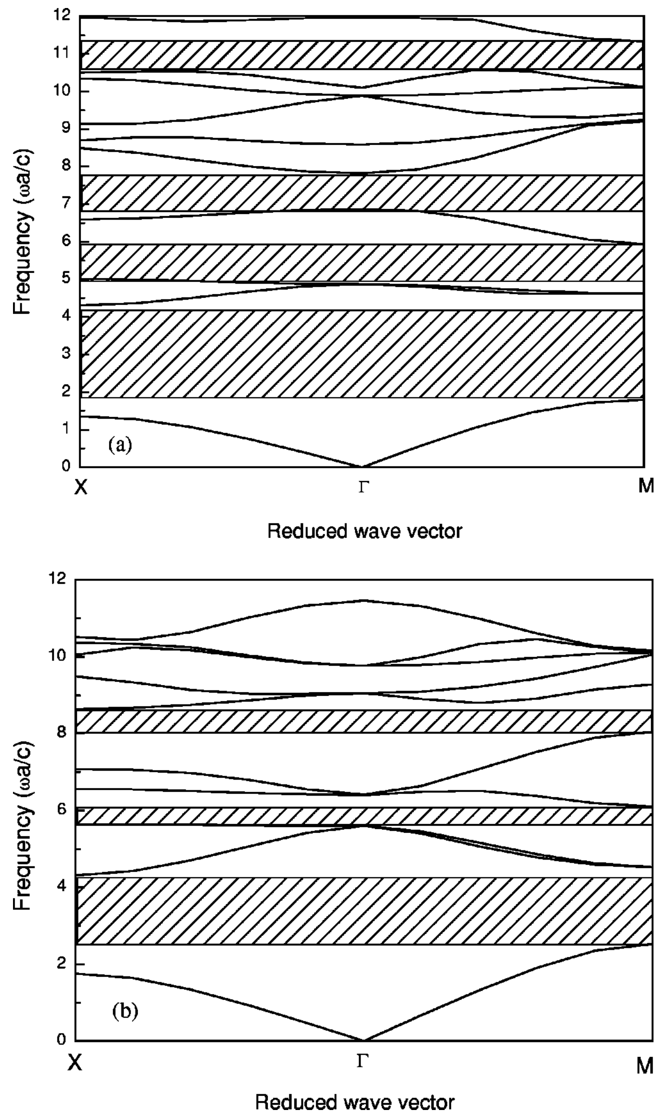

FIG. 4. Acoustic band gaps of water in mercury. The filling fraction is $F = 0.50$: (a) $\theta = 0°$ and (b) $\theta = 45°$. The hatched region represents a complete band gap.

A similar system of mercury rods in water, but with the circular cross section, has been reported [16], the results show that the largest band gap $\Delta\omega/\omega_g = 0.707$ appears at the close-packing value $F = 0.785$. So it is a very effective approach for us to obtain even large band gaps by the mechanism of rotating the square rods for the systems of high-density rods in low-density host.

As a comparison, we also investigate the case of water square rods in mercury host. The acoustic band structures are shown in Fig. 4(a) for $\theta = 0°$ and 4(b) for $\theta = 45°$ at the filling fraction of $F = 0.50$, respectively. There are four band gaps among the first ten bands for $\theta = 0°$ as shown in Fig.

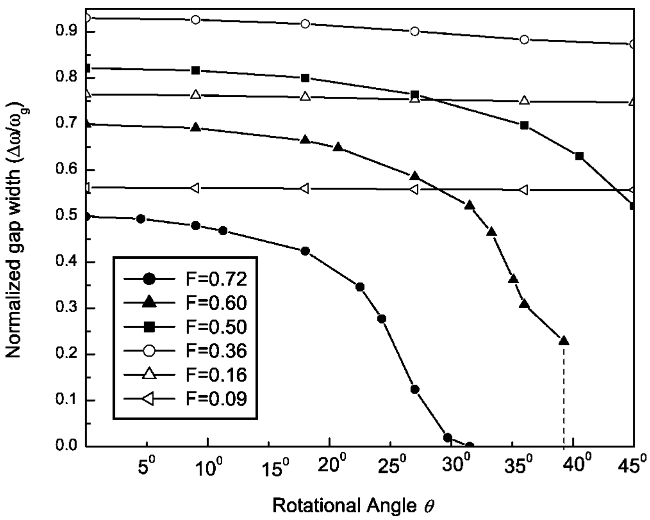

FIG. 5. Dependence of the normalized gap width of the acoustic band gap on the rotation angle for six different filling fractions $F=0.09$, 0.16, 0.36, 0.50, 0.60, and 0.73, respectively.

4(a), and the lowest gap, which is also the largest one, has gap width $\Delta\omega=2.51c/a$ and normalized gap width $\Delta\omega/\omega_g=0.82$. But for $\theta=45°$, as shown in Fig. 4(b), there are three band gaps among first ten bands, and the lowest gap, which is also the largest one, with gap width $\Delta\omega=1.79c/a$ and a normalized width $\Delta\omega/\omega_g=0.52$. This result shows that the gap width is narrower at $\theta=45°$ then at $\theta=0°$, which is different from the case of the mercury in water. In Fig. 5 we also have examined the variation of the normalized gap width with rotation angle θ for six different filing fraction of $F=0.09$, 0.16, 0.36, 0.50, 0.60 and 0.72, respectively. We have found that the normalized gap width will decrease monotonously as the rotation angle θ increases. Furthermore, the difference of the normalized gap width between $\theta=0°$ and $\theta=45°$ will be $\Omega\approx0.01$, 0.02, 0.06, 0.30, 0.47, and 0.50 for those six filling fractions, respectively. This infers that the effect of the rotation angle on the gap width will be gradually reduced as filling fraction F decreases.

IV. CONCLUSION

In summary, we have shown that the dependence of normalized gap width on the rotation angle in 2D liquid phononic crystals. In the systems of mercury square rods in the water host, i.e., the case of high-density rods in a low-density host, the acoustic band gaps enlarge gradually with increasing the rotation angle. For each filling fraction $F\leqslant0.50$, the maximum acoustic band gaps all appear at the same value $\theta=45°$, otherwise for filling fraction $F\geqslant0.50$, they are attained at angle θ_c, the largest angle dependent of the filling fraction. And the largest band gap is achieved at a high filling fraction $F=0.70$. In contrary, for the systems of water rods in mercury, that is the case of low-density rods in a high-density host, the acoustic band gaps narrow gradually with the filling fraction increasing. Because the ratio between the normalized gap width and rotation angle increases with the filling fraction for both mercury in water and water in mercury, it implies that the higher the filling fraction, the more evident the gap-tuning effect.

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